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Editor's Note

Just a reminder that we are eager to publish abstracts of all papers in the area of Decision Analysis, broadly conceived. The only requirements for our publishing an abstract of your work are:

1) That the paper itself be available for distribution upon request; and 2) that the abstract not exceed 200 words by much.

If there is a charge, please so indicate when you send your complete paper to the editor:

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Please phone or write in any changes in your activities or employment that could be of interest to our membership.

Please Note: Inform the ORSA business office of address changes; we get mailing labels from them! Thanks!

Inside

Departmental Editorship of Management Science.

With the burgeoning number of submissions to the Decision Analysis department of Management Science, Editor in Chief Don Morrison has appointed two Departmental Coeditors as joint successors to Robert L. Winkler: Gregory W. Fischer [Dept. of Social and Decision Sciences, Carnegie-Mellon University, Pittsburgh, PA 14213], and Irving H. LaValle [A. B. Freeman School of Business, Tulane University, New Orleans, LA 70118]. To prevent delays, papers in the Behavioral/ Experimental area should be sent to Greg Fischer, while papers in the Normative/Methodological area should be sent to Irv LaValle.

From the Chairperson

by Samuel E. Bodily; (804)924-4813; Box 6550, Charlottesville, VA 22906

The SIG council will discuss at the Vancouver meeting the possibility of a Publication Award for the SIG. The Ramsey Award provides recognition for distinguished contributions to the field of Decison Analysis (nominees for this award will be considered at the Fall 1989 New York meeting). There had been discussion in the past of having Theory and Application Wards in the SIG. The Application Award competition has received less than enthusiastic interest of late (for reasons not totally apparent). On the table at the SIG meeting will

(cont'd. page 2)

Chairperson, cont'd.

be discussion of and perhaps the Application Award as well as (which is one issue to discuss). Counsel member Robert Bordley is working on developing the idea. If you wish to comment on the award structure or the guidelines for such an award, please speak to Robert or any other officer.

Current officers of the SIG are, in addition to myself: Vice-Chair and Chair-elect: Robert L. Winkler (88-90); Secretary/Treasurer: Dennis M. Buede (88-90). Council: Robert T. Clemen (86-89), L. Robin Keller (86-89), Robert F. Bordley (87-90), Bruce R. Judd (87-90), Adam B. Borison (88-91), Irving H. LaValle (88-91).

Robin Keller has an excellent set of sessions organized for us at the Vancouver meeting.

Ramsey Medalists' Tape

The videotape of the presentations at the Denver meeting of the four Ramsey Medalists to date (Howard Raiffa, Ron Howard, Peter Fishburn, and Ward Edwards) is now available in VHS format. To purchase a copy, send your check, payable to ORSA, for $55.00 to: Ramsey Medalists Tape, Operations Research Society of America, Mount Royal and Guilford Avenues, Baltimore, MD 21202. An article summarizing these presentations is scheduled to appear in the April issue of OR/MS Today.

Authoring a Chapter in the Volume on Decision-Aiding Software and Decision Analysis

by Stuart S. Nagel, Dept. of Political Science, Lincoln Hall, 702 South Wright Street, University of Illinois, Urbana, IL 61801-3696

I shall greatly appreciate your informing me that you might like to publish a paper in the symposium volume on Decision-Aiding Software and Decision Analysis. It is planned as

Software, cont'd.

one of the volumes in a set on "Judgment and Decision-Making" edited by Kenneth Hammond as the series editor, probably to be published by John Wiley and Sons.

The essence of decision-aiding software in this context is any software that can process a set of (1) goals to be achieved, (2) alternatives available for achieving them, and (3) relations between goals and alternatives in order to choose or explain the best alternative, combination, allocation, or predictive decision-rule.

We would welcome papers that deal with decision-aiding software in such categories as (1) decision trees, (2) linear or mathematical programming, (3) statistical analysis, (4) spreadsheet-based software, (5) rule-based or expert-systems software, and (6) especially multi-criteria decision-making. Those are not mutually exclusive categories since one software package could fit into more than one category.

We are especially interested in papers that make comparisons among decision-aiding software packages. For example, we would welcome comparisons on such issues as the handling of multiple dimensions on multiple goals, missing information, allocation problems, multiple and possibly conflicting constraints, and the need for simplicity in drawing and presenting conclusions. papers that focus on a single software package are less acceptable, but the analysis should refer at least in a general way to other packages as well.

Examples of the kind of software we are interested in are given in such places as Patrick Humphreys and Ayleen Wisudha, Methods and Tools for Structuring and Analyzing Decision Problems (London School of Economics and Political Science, 1987);

Benjamin Radcliff, "Multi-Criteria Decision Making: A Survey of Software," 4 Social Science Microcomputer Review 38-55 (1986); and B. Golden, et al., "Decision Insight Systems for Microcomputers," in Saul Gass, et al., (eds.), Impacts of Microcomputers on Operations Research (North-Holland, 1986). Your name or firm may have been included in one of those sources.

The papers should be about 20 pages. They should be submitted before August 1, although there may be some flexibility in that regard. We would like this volume to be among the first two published in the series in 1989.

I look forward to receiving from you a one-page description of the kind of paper you would like to write for this volume. I especially look forward to receiving a good set of proposals and

papers and to seeing the papers developed, published, and well-received. Thank you for your participation in this important volume. Best wishes for its success in summarizing the stimulating new ideas concerning decision-aiding software. [**Editor's note**: Anyone interested in writing a Chapter for this volume should contact Professor Nagel at once concerning his press deadline, since he has indicated to me that the volume could go to press prior to August 1, but that he is also editing related publications relevant to decision-aiding software.

Bios of Council Nominees

F. B. Buoni

Fred Buoni has been Chairman of the Operations Research program at Florida Institute of Technology since he retired from the Air Force in 1979. From 1972 to 1979 he served as an adjunct professor at F.I.T. He received a Ph.D. in Nuclear Engineering from the Ohio State University in 1971 where he studied decision analysis with Bill Morris. Fred founded and is advisor of the ORSA Student Section at F.I.T. Although he has not served in any other position with ORSA, he has served other organizations including Sigma Xi, ANS, IIE, IEEE and the Florida Academy of Sciences.

Harvey J. Gold

Ph.D. University of Wisconsin, 1958. Senior Research Biochemist with U.S. Dept. Agriculture 1958-1963. Postdoctoral in Biomathematics 1963-65. Joined the Biomathematics Faculty at N.C. State Univ. in 1965 and served as Program Director 1977-1985. Member of the interdisciplinary Operations Research Program Committee at N.C. State since about 1975. Joined the Decision Analysis SIG of ORSA eight or nine years ago.

Research focus is on the use of simulation models as part of the basis for decision making. This involves development and application of methodology for combining information from simulation with relative frequency data and with subjective judgment. Applications to development of "intelligent" decision aids for agricultural and environmental management. Teaching is concentrated on two graduate level OR courses: System Modeling Theory and Decision Analytic Modeling. Consulting activity includes local industrial and human service organizations and conducting workshops on strategic planning and decision making.

Allan H. Murphy

Allan Murphy is professor of atmospheric sciences and statistics at Oregon State University. He received a B.S. degree (Meteorology) from M.I.T. and M.S. (meteorology), M.A. (mathematics/statistics), and Ph.D. (atmospheric and oceanic science) degrees from the University of Michigan. His research interests are focused primarily on applications of statistical and operations research methodology in atmospheric sciences. Dr. Murphy (or Allan) has undertaken numerous studies related to probability assessment and applied decision analysis, especially in the context of weather forecasting. He has held a variety of positions at universities and research institutes in the U.S. and Europe and has published extensively in the meteorological literature, as well as in the literature of operations research, psychology, and statistics.

Ross D. Shachter

Ross D. Shachter is an assistant professor in the Department of Engineering-Economics Systems at Stanford University. He has a PhD in Operations Research from the University of California, Berkeley. His research has been mainly in decision analysis, with emphases on medical decision making and on the representation and analysis of models with influence diagrams. He has been an active participant in the workshops on Uncertainty in Artificial Intelligence: he was program chair for the fourth workshop and is general chair for the upcoming fifth workshop. During the two academic years 1986-8, he was a

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visiting professor at the Center for Health Policy at Duke University, where he was working on methodology for medical technology assessment.
Shachter (cont'd.)

His papers have appeared in Operations Research, Management Science, and Medical Decision Making, and he developed DAVID: Influence Diagram Processing System for the Macintosh.

he is a member of both ORSA and TIMS and the Society for Medical Decision Making.

Abstracts of Papers Received

From **Farrokh Alemi**, Jefferson Medical College, 1025 Walnut Street, Room 119, Philadelphia, PA 19107:

Balance in Bayesian Models: Implications for Decision Analysis

This paper shows how a multi-dimensional Bayesian probability model can be constructed through querying an expert. It suggest how the analyst could and should categorize the factors described by the expert into pro and con factors. Next, it presents a formula for assessing whether pro factors exceed con factors in a manner that is counter intuitive to the expert. No data about successful application of this theory are included. This paper presents instead the theoretical foundations from which these procedures follow, and describes the implications of this theory for decision analysis, for foundation of statistics, and for design of expert systems. The paper argues that Bayesian statisticians are wrong in assuming that the rule for stopping to gather information has no impact on Bayesian predictions and Pearson-Neyman statisticians are wrong in assuming that in their experiments they ignore prior odds.

From **Farrokh Alemi**, Jefferson Medical College, 1025 Walnut Street, Room 119, Philadelphia, PA 19107, **Flora Cherry**, and **Greg Meffert**, both at Health Systems Management Dept., School of Public Health and Tropical Medicine, Tulane Medical Center, 1430 Tulane Avenue, New Orleans, LA 70112:

Rehearsing Decisions May Help Teens: An Evaluation of a Simulation Game

This paper presents a new approach to preventing adolescent pregnancy. Information, alone is not sufficient to prevent teen pregnancy. The teenagers ability to choose and remain committed to a decision also needs to be developed. Because decision making skills are best learned through practice in an environment with frequent feedback, we have developed a computer game which simulates the consequences of different sexual roles. In addition, the game is intended to increase communication about sex between teenagers and their role models (peers, teachers and/or parents). Increased communication is expected to reduce the feeling of guilt and lead to either consistent abstention from sex or consistent contraceptive use. The paper reports on the development of the computer game and the preliminary evaluation of its impact.

From **Robert G. Batson** and **Janet E. Walker**, Industrial Engineering Department, University of Alabama, Box 870288, Tuscaloosa, AL 35487-0288:

Estimating Beta Distribution Shape Parameters From Three-and-Four-Point Fractile Encoding

Beta distributions are widely used to model continuous probability distributed over a finite interval. Many decision analysis and probabilistic risk analysis applications depend on elicitation of expert opinion, encoded as a beta distribution, as one critical type of input. All major simulation packages now

in use provide the beta input option, and require the analyst to input shape parameters. This paper describes how these parameters may be estimated from three-point or four-point estimates (the mode or the median, or both, and a pair of p-fractiles) obtained from interviews with experts. Eighteen methods for such parameter estimation are identified, and a comprehensive test of these methods using 91 test distributions is reported. Error analysis is performed and the best three-point methods to estimate the shape parameters based on the mode and median, respectively, are identified. Seven four-point approximations are also studied and compared with each other and the three-point methods.

From **Robert T. Clemen**, College of Business Administration, University of Oregon, Eugene, OR 97403, and **Robert L. Winkler**, Fuqua School of Business, Duke University, Durham, NC 27706:

Unanimity and Compromise Among Probability Forecasters

When two forecasters agree regarding the probability of an uncertain event, should a decision maker adopt that probability as his or her own? A decision maker who does so is said to act in accord with the unanimity principle. We examine a variety of Bayesian consensus models with respect to their conformance (or lack thereof) to the unanimity principle and a more general compromise principle. In an analysis of a large set of probability forecast data from meterology, we show how well the various models, when fit to the data, reflect the empirical pattern of comformance to these principles.

From **Samuel Holtzman**, Strategies Division Group, 2440 Sand Hill Road, Mendo Park, CA 94025-6900:

Decision Analysis and Influence Diagrams: Fundamental Concepts and Glossary

Decision analysis (DA) comprises the philosophy, methodology, and professional discipline necessary for helping individuals make important decisions. To this end, decision analyst-particularly those associated with the "Stanford School" of DA-have painstakingly developed a clear and precise terminology describing both the process and content of a decision analysis. Within this terminology, two concepts-the *decision analysis cycle* and *influence diagrams*-are particularly important and play a central role in DA theory and practice. Following a brief discussion of these two concepts in terms of their attention-focusing and insight-producing effects, this paper defines key decision analysis and influence diagram terms as a cross-referenced glossary.

From **Ronald A. Howard**, Department of Engineering - Economic Systems, Stanford University, Stanford, CA 94305:

From Influence to Relevance to Knowledge

Over the years, experience with the influence diagram has shown that it is an effective means for communication with both decision makers and computers. The influence diagram has proved to be a new "tool of thought" that can facilitate the formulation, assessment, and evaluation of decision problems. Practical use has provided several refinements and extensions of the concept that increase

effectiveness.

Refinements include the following developments. Since the arrows between uncertainties in the diagram represent an assessment order rather than a chain of physical effects, we prefer the term "relevance" to describe the relationship. Then it is clear that if A is relevant to B, B is relevant to A. The term influence often carries a causal connotation which is not appropriate. We have also learned the importance of the clarity test to assure that the quantities assessed in the diagram are clearly defined. If we desire to be able to calculate the value of clairvoyance on any uncertain quantity in the diagram, then we must draw it in canonical form. In canonical form, there must be no arrow from a decision node to a chance node.

Some extensions of the influence diagram ideas arise when we focus on diagrams that contain only chance nodes, which we call "relevance diagrams". When we use a relevance diagram to assess the information that an individual or a group has about a set of uncertain quantities, we speak of the result as a "knowledge map." Redundant knowledge maps allows us to assess information in ways that do not correspond to any assessment order. Disjoint knowledge maps permit us to reduce the amount of assessment to that required for the decision problem by forgoing the opportunity to ask about other probabilistic results. The distinction between assessed and evocative knowledge maps allows us to think explicitly of the many factors that could affect an assessment without becoming committed to numerical specification.

These new developments in influence diagrams offer the promise of an even wider range of practical use by both analysts and decision-makers.

From **Marie Foley Kijewski, Richard G. Swenson,** and **Philip F. Judy,** Department of Radiology, Bingham and Women's Hospital, 75 Francis Street, Boston, MA 02115:

Analysis of Rating Data From Multiple Alternative Tasks

A standard method of estimating a single ROC curve from rating data for two stimulus alternatives has been extended to ratings of multiple alternatives. An observer's ratings are assumed to represent ordinal classifications of a unidimensional decision variable that has a separate distribution for each of M possible stimuli. From these rating data, a maximum-likelihood procedure simultaneously estimates the rating-category boundary values and the 2 (M-1) distribution parameters that specify ROC curves between all pairs of the M stimulus alternatives. Many stimulus manipulations, particularly those investigated in psychological experiments with visual or auditory stimuli, could justify this M-alternative rating procedure and analysis. An advantage of this method is that it allows reliable measurement of an observer's performance indices at much higher values than does the two-alternative method.

The assumption of a unidimensional decision variable may be too restrictive for general decision-making situations, where the decisions among alternatives often involve multiple sources of information. However, the two-alternative method is commonly used to fit rating-ROC curves for some decision-making tasks, such as diagnosis from medical images, for which the multiple-alternative procedure actually might be more appropriate.

From **Don N. Kleinmuntz**, Sloan School of Management, Massachussetts Institution of Technology, E 52-568, 50 Memorial Drive, Cambridge, MA 02139:

Decomposition and the Control of Error in Decision Analytic Models

Decision analytic models rely upon the general principle of problem decomposition: Large and complex decision problems are reduced to a set of relatively simple judgments and these component judgments are then combined using mathematical rules derived from normative theory. This paper discusses the value of decomposition as a procedure for improving the consistency of decision making. Various definitions of error and consistency are discussed. Linear decomposition models are argued to be particularly useful for the control of random response errors in the component judgments. Implications for decision analysis research and practice are considered, and decision makers' evaluations of the costs and benefits of decision analysis are discussed.

From **Don N. Kleinmuntz**, Sloan School of Management, Massachussetts Institution of Technology, E 52-568, 50 Memorial Drive, Cambridge, MA 02139 and **David A. Schkade**, Department of Management, Graduate School of Business, University of Texas, Austin, Texas 78712:

The Cognitive Implications of Information Displays in Computer-Supported Decision Making

A theory-based approach for research on information displays in computer-supported decision making is proposed. Information display characteristics can influence the decision maker's selection of a cognitive strategy. Since the effectiveness of decision making depends, in part, on the strategy selected, knowledge about the display-strategy relationship can ultimately improve the quality of decision support by identifying displays that encourage the selection of effective strategies. Support for this approach is provided by a discussion of psychological research on strategy selection, focusing on cognitive effort and the accuracy of decision strategies as components of a cognitive incentive system for decision makers. The results of empirical research on four general aspects of information displays are reviewed: the form and features of individual data items, the organization of individual display items into patterns, changes in displays over time, and the degree of flexibility the decision maker is permitted in determining display characteristics. Proposed directions for information display research are discussed.

From **Vahid Lotfi***, **Theodore J. Stewart,**** and **Stanley Zionts***, * = Department of Management Science and Systems, School of Management, State University of New York at Buffalo, Buffalo, NY 142650; ** = Department of Mathematical Statistics, University of Capetown, Cape Town, South Africa:

An Aspiration - Level Interactive Model for Multiple Criteria Decision Making

A simple, eclectic approach for solving discrete alternative multiple criteria decision problems is presented. It is based on the concept of the level of aspiration, and draws on ideas of various researchers. It assumes that the user has a set of alternatives with each alternative having a score on each of a number of objectives or measures of performance. The user determines his levels of aspiration for different objectives in an interactive personal computer environment in which he is given considerable feedback as to the degree of feasibility of each level of aspiration as well as the degree of feasibility with respect to all levels of aspiration as a whole. The closest nondominated solution to the solution specified by the levels of aspiration is provided, as are other useful outputs. Our objective in the paper is to develop a method based on these ideas that is easy to use and easy to understand. We have implemented the approach on a personal computer (i.e., an IBM PC or compatible with 256K RAM). We describe an experimental application in which 40 students in an M.B.A. program used the method to select a computer to purchase. An example is included in the appendix.

From **James E. Matheson**, Strategic Decisions Group, 2440 Sand Hill Road, Menlo Park, CA 94025-6900:

Using Influence Diagrams To Value Information and Control

The well-known concept of the value of perfect information and the recent concept of the value of control are very useful in gaining insight about decision situations. However, using the familiar tool of decision trees for this purpose can be confusing and misleading. This paper shows how influence diagrams can clarify and correctly pose value of information questions to evaluate new opportunities for gathering information, uses the concept of the *clairvoyant* to meaningfully assess the required joint probability distributions, and introduces the value of control as a way to gain insight into the usefulness of generating new alternatives that enable more control over uncertain variables.

From **H.V. Ravinder**, Anderson School of Management, University of New Mexico, Alberqueque, NM 87131:

Probability Decompositions With Biased Elicitations

This paper deals with the question of bias in the elicitations used to obtain a decomposition estimate of a subjective probability. Biases in the input elicitations introduce a bias into the final estimate; the direction and magnitude of this bias depend on the input biases. Under certain circumstances the bias in the decomposition estimate is substantially smaller than the bias in a direct estimate. Thus given the right conditions, decomposition may be used as a bias reduction technique. But bias in the input elicitations also affects the error variance of the decomposition estimate, usually adversely. A balance has to be struck between bias reduction requirements and random error reduction requirements.

From **John D. Sterman**, E 52-562, Sloan School of Management, Massachussetts Institute of Technology, 50 Memorial Drive, Cambridge, MA 02139:

Deterministic Chaos in an Experimental Economic System

An experiment with a simulated macroeconomic system demonstrates that the decision-making processes of agents can produce deterministic chaos. Subjects managed capital investment in a simple multiplier-accelerator economy. Performance, however, was systematically suboptimal. A model of the subjects' decision rule is proposed and related to prior studies of dynamic decision making. Econometric estimates show the model is an excellent representation of the actual decisions. The estimated rules are then simulated to evaluate the stability of the subjects' decision processes. While the majority of the estimated rules are stable, approximately 40% yield a variety of dynamics including limit cycles, period multiples, and chaos. Analysis of the parameter space reveals a complex bifurcation structure. Implications for models of human systems and experimental studies of economic dynamics are explored.

Misperceptions of Feedback in Dynamic Decision Making

In recent years laboratory experiments have shed significant light on the behavior of economic agents in a variety of microeconomic and decision-theoretic contexts such as auction markets, portfolio choice, and preference elicitation. Despite the success of experimental techniques in the micro domain, there has been relatively little work linking the behavior of decision makers to the dynamics of larger organizations such as corporations, industries or the macroeconomy. This paper presents a laboratory experiment in which subjects manage a simulated economy. Subjects must invest sufficient capital plant and equipment to satisfy demand. Subjects were given complete and perfect information regarding the structure of the simulated economy, the values of all variables and the past history of the system. Nevertheless, the overwhelming majority of the subjects generate significant and costly oscillations. A simple decision rule based on the anchoring and adjustment heuristic is shown to simulate the subjects' decisions quite well. Several distinct sources of the subjects' poor performance are identified and termed "misperceptions of feedback." The decision rule is related to various models of economic fluctuations; implications for experimental investigation of dynamic decision making in aggregate systems are explored.

From **John D. Sterman**, E 52-562, Sloan School of Management, Massachussetts Institute of Technology, 50 Memorial Drive, Cambridge, MA 02139 and **Erik Mosekilde** and **Erik Larsen**, both at Physics Laboratory III, Technical University of Denmark, DK-2800 Lyngby, Denmark:

Experimental Evidence of Deterministic Chaos in Human Decision-Making Behavior

An experiment with a simulated microeconomic system demonstrates that the decision-making process of human subjects can produce deterministic choas. Participants managed a commodity production-distribution system to minimize costs. Performance, however, was systematically suboptimal. A model of the subjects' decision rule is proposed. Econometric estimates show the model is an

excellent representation of the actual decisions. Simulation of the estimated rules yields, stable, periodic, quasiperiodic, and chaotic solutions. Analysis of the parameter space reveals a complex structure including mode-locking, a devil's staircase, and fractal basin boundaries. Implications for modeling human systems are explored.

From **Peter P. Wakker**, Department of Mathematical Psychology, University of Nijmegen, P. O. Box 9104, 6500 HE Nijmegen, The Netherlands:

Additive Representations of Preferences:
A New Foundation of Decision Analysis

This book (just published by Kluwer Academic Publishers) presents a new foundation of decision analysis. Starting point is a new foundation for subjective expected utility maximization which, contrary to the usual foundation of decision analysis, does not need any lotteries or 'objective', 'given', probabilities, by generalizing the contributions of von Neumann & Morgenstern, and Anscombe & Aumann. The main tool in the derivations is a tradeoff idea from multiattribute-utility theory, leading to the result that subjective expected utility is appropriate if and only if no 'contradictory tradeoffs on consequences' are revealed. From the introduction we quote:

'It is common use in economic analyses that scientists, using subjective expected utility without lotteries available, for a justification refer to Savage (1954, Foundations of Statistics). We are however not aware of an economic analysis in which actually the restrictive conditions of Savage (1954) are verified. The restrictive condition of our set-up, continuity of utility, usually is satisfied.'

Chapter I shows how to relate ('revealed') preferences to choice making, Chapter II introduces the tradeoff-idea from multiattribute-utility theory and studies implications of the 'sure-thing principle'. Chapter III gives a self-contained presentation of additive representations of preferences, made accessible through many illustrations. Chapter IV gives the 'Central Theorem' of the book, showing that SEU is appropriate if and only if no contradictory tradeoffs on consequences are revealed. Chapter V generalizes this result. For instance, contrary to Savage's result, our result does not require boundedness of utility. Chapter VI gives an introduction into the nowadays popular approach which deals with probabilities in a nonlinear way, enabling the incorporation of optimism and pessimism in a way excluded by expected utility. Chapter VII shows that in the new approach classical results concerning risk aversion can still be obtained. As compared to the work of Pratt and Arrow, not any quantification, given in advance, is needed. No probabilities have to be known, and no quantifications of the consequences that may result from decisions are needed. In the Appendix, a concise presentation is given of the 'classical' derivations of expected utility.

From **William F. Webster**, Department of Decision and Information Sciences, University of Florida, Gainesville, FL 32611:

Nonlinear Probability Functions in Models of Choice Behavior:
Evidence, Implications, and Measurement

Research suggests that decision makers consistently violate various conditions of the expected utility hypothesis. One of the most notable characteristics of the empirical studies of choice behavior is the systematic finding of a nonlinear probability function. Such a property is not an acceptable condition in the classical models of expected utility. This paper reviews the previous empirical evidence that supports the existence of nonlinear probability functions, and discusses the implications for decision makers. It also presents a methodology for the measurement of the characteristic in a prospect-based model of choice behavior. Empirical evidence is presented that supports the general existence of the nonlinear probability functions for many subjects in models of choice behavior.

An External Utility Risk Model: A Model of Preferences and a Measure of Risk Based on Observed Choice Behavior

This study develops a utility model designed to reflect choice behavior accurately by incorporating as conditions in a normative development certain behavioral characteristics commonly exhibited by decision makers in empirical studies. There are three particular characteristics that decision makers consistently exhibit that are not usually associated with most normative models of expected utility theory, notably: (1) the existence of a reference outcome; (2) a nonlinear function on probabilities; and (3) a risk attitude that is generally risk averse for gains and risk seeking for losses. The final numerical representation of the model includes a probability term representing statistical expectation, a two-piece utility function on outcomes, and an additional function on probability. This additional function is compared to the concept of relative risk aversion and interpreted as a measure of individual risk attitude. The model developed in this paper is referred to as the expected utility risk model. An exploration of the empirical implications of the model provides insight into a new method for measuring individual risk attitudes, called probability-risk, that is based on differing levels of probability in risky situations rather than the shope of the utility function as used in traditional models. The model can be shown to be similar to other models designed to explain axiom violations and aberrations of the expected utility hypothesis.